This question paper contains 7 printed pages] (3)

Roll No. | | | | | | | | | | | |

2019

S. No. of Question Paper : **2248**

Unique Paper Code : **32351201**

Name of the Paper : **Real Analysis**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

There are internal choices in Q. Nos. **2-5**.

1. Prove or disprove : 6×2½=15

(*a*) If $x \in R$, $x > 0$, then $\frac{1}{x} > 0$.

(*b*) If s is an upper bound of a non-empty set S such that $s \in S$, then $s = \sup S$.

(*c*) A sequence (x_n) satisfying $\lim (|x_{n+1} - x_n|) = 0$ is convergent.

P.T.O.

(*d*) $\lim ((a^n + b^n)^{1/n}) = b$, where $0 < a < b$.

(*e*) The series $\sum_{n=1}^{\infty} (\cos nx)$ converges for all $x \in \mathbf{R}$.

(*f*) $\sum_{n=1}^{\infty} \frac{n2^n}{(n^2+1)}$ is a convergent series.

2. Answer any *three* parts : 3×5=15

(*a*) State and prove the Density Theorem for real numbers.

(*b*) (*i*) Let $a, b \in \mathbf{R}$ and suppose that for every $\varepsilon > 0$, we have $a \leq b + \varepsilon$. Show that $a \leq b$.

(*ii*) Let S be a non-empty subset of $\mathbf{R}$. Show that $u \in \mathbf{R}$ is an upper bound of S if and only if the conditions $t \in \mathbf{R}$, $t > u$ imply $t \notin S$.

(*c*) Let S be a non-empty bounded set in $\mathbf{R}$ and let $b < 0$. Prove that $\inf(bS) = b(\sup S)$ and $\sup(bS) = b(\inf S)$.

(*d*) If S is a non-empty subset of $\mathbf{R}$, show that S is bounded if and only if there exists a closed and bounded interval I of $\mathbf{R}$ such that $S \subseteq I$.

3. Answer any *three* parts : 3×5=15

(*a*) Find the limit of the following sequences whose *n*th term is given by :

(*i*) $x_n = \frac{n}{b^n}$, where $b > 1$

(*ii*) $y_n = \frac{\sin n}{n} + \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)$.

(*b*) Prove that if a sequence (x_n) is increasing and bounded above, then it converges to u where u is the least upper bound of the set $\{x_n : n \in \mathbf{N}\}$.

(*c*) If a sequence (x_n) of real numbers converges to a real number x, prove that every subsequence (x_{n_k}) of (x_n) converges to x.

(*d*) State the Cauchy Convergence Criterion for sequences. Use it to show that the sequence (x_n) defined by

$$x_n = \frac{1}{1!} - \frac{1}{2!} + \ldots\ldots + \frac{(-1)^{n+1}}{n!}$$

is convergent.

4. Answer any *three* parts : 3×5=15

(*a*) Use the integral test to check the convergence of the series :

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}.$$

(*b*) When do we say that the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent ? Show that the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(\sqrt{n+1} - \sqrt{n}\right)$$

is absolutely convergent.

(*c*) Test the convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$

(*ii*) $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}.$

(*d*) (*i*) Find all $x \in R$ for which the series $\sum_{n=1}^{\infty} e^{nx}$ converges.

(*ii*) Show that the series $\sum_{n=1}^{\infty} \log\left(\frac{n}{n+1}\right)$ is divergent.

5. (*a*) (*i*) Let X and Y be non-empty sets and let $h : X \times Y \to \mathbf{R}$ have bounded range in $\mathbf{R}$. Let $f : X \to \mathbf{R}$ and $g : Y \to \mathbf{R}$ be defined by

$$f(x) = \sup \{h(x, y) : y \in Y\},$$

$$g(y) = \inf \{h(x, y) : x \in X\}.$$

Prove that :

$$\sup \{g(y) : y \in Y\} \leq \inf \{f(x) : x \in X\}.$$

(*ii*) Give an example of a set which has exactly two limit points. 4,1

Or

(*i*) Show that for any real numbers p, q and rational number r such that $r < p + q$, there exist rational numbers $r_1 < p$ and $r_2 < q$ such that $r = r_1 + r_2$.

(*ii*) Provide a bijection between $\mathbf{N}$ and the set of all odd integers greater than 49. 3,2 (*b*) (*i*) If $\lim (x_n) = x(\neq 0)$, prove that there is a positive number A and a natural number N such that $|x_n| > A$ for all $n \geq N$.

(*ii*) Is the sequence (x_n) where

$$x_n = \frac{n^3 + 3n^2}{n+1} - n^2$$

bounded ? Justify. 3,2

Or

Is the sequence (x_n) where

$$x_n = \frac{n^2}{n^3 + n + 1} + \frac{n^2}{n^3 + n + 2} + \ldots\ldots + \frac{n^2}{n^3 + 2n}$$

convergent ? If yes, find its limit. 5

(*c*) State the Alternating Series Test. Show that the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$$

is conditionally convergent. 5

Or

If the series $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ are convergent, then prove that the series

$$\sum_{n=1}^{\infty} a_n b_n$$

is convergent where $a_n \geq 0$ and $b_n \geq 0$ for all $n \in \mathbf{N}$.

Hence or otherwise show that the series $\sum_{n=1}^{\infty} \frac{a_n}{n}$ is convergent whenever $\sum_{n=1}^{\infty} a_n^2$ is convergent. 5

This question paper contains 8 printed pages] (4)

Roll No. | | | | | | | | | | | |

2019

S. No. of Question Paper : **2249**

Unique Paper Code : **32351202** **IC**

Name of the Paper : **Differential Equations**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use of non-programmable scientific calculators is allowed.

Section–I

1. Attempt any *three* parts. Each part is of **5** marks.

(*a*) Solve the initial value problem :

$$(x^2+1)\frac{dy}{dx}+4xy=x,\ y(2)=1.$$

(*b*) Determine the constant A in the following equation such that the equation is exact, and solve the resulting exact equation :

$$(x^2+3xy)dx+(Ax^2+4y)dy=0.$$

(*c*) Solve the differential equation :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=4x.$$

(*d*) Solve the differential equation :

$$2xy\frac{dy}{dx}=x^2+2y^2.$$

2. Attempt any *two* parts. Each part is of **5** marks.

(*a*) A cylindrical tank with length 5 ft and radius 3 ft is situated with its axis horizontal. If a circular bottom hole with a radius of 1 inch is opened and the tank is initially half full of xylene, how long will it take for the liquid to drain completely ?

(*b*) Suppose that sodium pentobarbital is used to anesthetize a dog. The dog is anesthetized when its bloodstream contains at least 45 milligrams (mg) of sodium pentobarbital per kilogram of the dog's body weight. Suppose also that sodium pentobarbital is eliminated exponentially from the dog's bloodstream, with a half life of 5 hours. What single dose should be administered in order to anesthetize a 50 kilogram dog for 1 hour ?

(*c*) Suppose that a motorboat is moving at 40 ft/sec when its motor suddenly quits, and that 10 seconds later the boat has slowed to 20 ft/sec. Assume that the resistance it encounters is proportional to its velocity. How far will the boat cast in all ?

Section–II

3. Attempt any *two* parts. Each part is of **7.5** marks.

(*a*) Consider the American system of two lakes : Lake Erie feeding into Lake Ontario. Assuming that volume in each lake to remain constant and that Lake Erie is the only source of pollution for Lake Ontario.

(*i*) Write down a differential equation describing the concentration of pollution in each of two lakes, using the variables V for volume, F for flow, $c(t)$ for concentration at time t and subscripts 1 for Lake Erie and 2 for Lake Ontario.

(*ii*) Suppose that only unpolluted water flows into Lake Erie. How does this change the model proposed ?

(*iii*) Solve the system of equations to get expression for the pollution concentration $c_1(t)$ and $c_2(t)$.

(*b*) In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X.$$

(*i*) Find the non-zero equilibrium population.

(*ii*) At what critical harvesting rate can extinction occur ?

(*c*) Consider the population of the country. Assume constant per capita birth and death rates and that the population follows an exponential growth (or decay) process. Assume there to be significant immigration and emigration of people into and out of the country.

(*i*) Assuming the overall immigration and emigration rates are constant, formulate a single differential equation to describe the population size over time.

(*ii*) Suppose instead that all immigration and emigration occurs with a neighbouring country, such that the net movement from one country to the another is proportional to the population difference between the two countries and such that people move to the country with the larger population. Formulate a coupled system of equations as a model for this situation.

Section–III

4. Attempt any *four* parts. Each part is of **5** marks.

(*a*) Find general solutions (for $x > 0$) of the Euler's equation :

$$x^2 y'' + 7xy' + 25y = 0.$$

(*b*) Solve the initial value problem by using the method of undetermined coefficients :

$$y'' + y = \sin x; \quad y(0) = 0, y'(0) = -1.$$

(*c*) Use the method of variation of parameters to find the solution of the differential equation :

$$y'' + 3y' + 2y = 4e^x.$$

(*d*) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -10$ m/s. Find the amplitude, period, and frequency of the resulting motion.

(*e*) A body of mass $m = 2$ kg is attached to both a spring with a spring constant $k = 4$ and a dashpot with a damping constant $c = 3$. The mass is set in motion with initial position $x_0 = 2$ and initial velocity $v_0 = 0$. Find the position function $x(t)$ and determine whether the motion is overdamped, critically damped or underdamped. If it is underdamped, find its pseudofrequency, pseudoperiod of oscillation and its time varying amplitude.

Section–IV

5. Attempt any *two* parts. Each part is of **7.5** marks.

(*a*) Consider a simple model for a battle between two armies. Assumed that the probability of a single bullet hitting its target is constant. Suppose that the soldiers from the red army are visible to the blue army. But the soldiers from the blue army are hidden.

(*i*) Develop the model for describing the rate of change of number of soldiers in each of the armies.

(*ii*) By making appropriate assumptions, extend the model to include the reinforcements if both of the armies receive reinforcements at constant rates.

(*b*) Consider a disease where all those who are infected remain contagious for life. Assume that there are no births and deaths.

(*i*) Write down suitable word equations for the rate of change of number Susceptible and Infective and hence develop a pair of differential equations.

(*ii*) Use the chain rule to find a relationship between the number of susceptibles and the number of infectives.

(*iii*) Draw a sketch of typical phase-plane trajectories. Deduce the direction of travel along the trajectories providing reasons.

(c) A model of a three species interaction is :

$$\frac{dX}{dt} = a_1 X - b_1 XY - c_1 XZ,$$

$$\frac{dY}{dt} = a_2 XY - b_2 Y,$$

$$\frac{dZ}{dt} = a_3 XZ - b_3 Z.$$

Where $a_i\ b_i\ c_i$ for $i = 1, 2, 3$ are all positive constants. Here $X(t)$ is the prey density and $Y(t)$ and $Z(t)$ are the two predator species densities.

(i) Find all possible equilibrium populations. Is it possible for all three populations to coexist in equilibrium ?

(ii) What does this suggest about introducing an additional predator into an ecosystem ?

Sl. No. of Q.P. : 3656 (5) 14/5/19

Unique Paper Code : 235201

Name of Paper : Differential Equations and Mathematical Modeling –I (MAHT-201)

Name of Course : B.Sc.(H) Mathematics

Semester : II

Duration : 3 hours

Maximum Marks : 75 Marks

Instructions for Candidates:

1. Attempt all questions.
2. Use of non-programmable scientific calculator is allowed.

Section-I

Q1. ***Attempt any three of the following:*** *(5+5+5)*

(a) Verify that the differential equation $(e^x \sin y + \tan y)dx + (e^x \cos y + x\sec^2 y)dy = 0$ is exact and hence solve it.

(b) Solve the differential equation : $(x^3+1)\frac{dy}{dx} + 6x^2 y = 6x^2.$

(c) Solve the initial value problem:

$$(x+2y+3)dx + (2x+4y-1)dy = 0, y(0) = 1.$$

(d) Solve the differential equation $x^2\frac{dy}{dx} = 1 - x^2 + y^2 - x^2y^2.$

gap

Q2. ***Attempt any two of the following:*** *(5+5)*

(a) A motorboat starts from rest (initial velocity $v(0) = v_0 = 0$). Its motor provides a constant acceleration of 4 ft/s^2, but water resistance causes a deceleration of $\frac{v^2}{400} ft/s^2$. Find v when t =10 s, and also find the limiting velocity as $t \to +\infty$ (that is, the maximum possible speed of the boat).

(b) A4- lb roast, initially at 50° F oven at 5:00 P.M. After 75 minutes it is found that the temperature $T(t)$ of the roast is 125° F. When will the roast be 150° F?

(c) A certain city had a population of 25000 in 1960 and a population of 30000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2000.

Section-II

Q3. ***Attempt any two of the following:*** *(7.5+7.5)*

(a) A public bar opens at 6:00 pm and is rapidly filled with clients of whom the majority are smokers. The bar is equipped with ventilators which exchange the smoke- air mixture with fresh air. Cigarette smoke contains 4% carbon monoxide and a prolonged exposure to a concentration of more than 0.008% can be fatal. The bar has a floor area of 10m by 15m and a height of 4m. It is estimated that smoke enters the room at a constant rate of 0.006% m^3/min and that the ventilators remove the mixture of smoke and air at 10 times the rate at which smoke is produced.

Develop a differential equation system that describes this process, defining all variables and parameters as required. Establish the time when the lethal limit is reached.

(e) Show first that the three solutions, $y_1 = x, y_2 = x\log x$ and $y_3 = x^2$ of the third order differential equation $x^3y^{(3)} - x^2y'' + 2xy' - 2y = 0$ are linearly independent on the open interval $x > 0$. Then find a particular solution that satisfies the initial conditions $y(1) = 3, y'(1) = 2, y''(1) = 1$.

Section-IV

Q5. *Attempt any two of the following:* *(7.5+7.5)*

(a) Consider a battle between two armies (red and blue) where blue army uses aimed fire and red army uses random fire. Assume that the red army has a significant loss due to disease, where associated death rate is proportional to the number of soldiers in that army, and this army receives reinforcement at a constant rate. Starting from a compartmental diagram and introducing all the variables and parameters needed, formulate a model for this battle.

(b) Determine a compartmental diagram and appropriate word equation for each of the two populations, the predator and the prey. Let *X(t)* denotes the number of prey per unit area and *Y(t)* the number of predators per unit area. Assume that the per-capita birth rate for the prey is the per-capita death rate of prey due to being killed by the predators will depend on the predator density, the simplest assumption is to assume this per-capita rate is proportional to the predator density, a per-capita rate c_1Y. Assume a_2 to be constant per-capita death rate for the predators independent of the prey density. Also assume that the prey are an essential requirement for births of the predator, so the per-capita birth rate for the predators will be the sum of a natural rate, a constant , say, b_2 plus an additional rate which is proportional to the rate of prey killed. Using these assumptions and the word equations, formulate differential equations for the prey and predators densities.

(c) In a long range battle, neither army can see the other, but fires into a given area. A simple mathematical model describing this battle is given by the coupled differential equations

$$\frac{dR}{dt} = -c_1RB\,, \qquad \frac{dB}{dt} = -c_2RB$$

where c_1 and c_2 are positive constants.

(i) Use the chain rule to find a relationship between R and B, given the initial number of soldiers for the two armies are r_0 and b_0 respectively.

(ii) Draw a sketch of typical phase-plane trajectories.

(iii) Explain how to estimate the parameter c_1 given that the blue army fires into a region of area A.

(b) Write down the word equations which describe the movement of the drug between the two compartments in the body, GI-tract and the bloodstream, when a patient takes a single pill.

(i) From the word equations, develop the differential equation system which describes this process, defining all variables and parameters as required.

(ii) The constants of proportionality associated with rates at which the drug diffuses from GI-tract into the bloodstream and then is removed from the bloodstream, are 0.72 $hour^{-1}$ and 0.15 $hour^{-1}$ respectively. Suppose initially the amount of drug in GI-tract is 0.0001 mg and none in the bloodstream. Determine the level of drug in the bloodstream after 6 hours.

(c) Consider the harvesting model

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h.$$

(i) Find two non-zero equilibrium populations.

(ii) If the harvesting rate h is greater than some critical value h_c, the non-zero equilibrium values do not exist and the population tends to extinction. What is this critical value h_c?

(iii) If the harvesting rate is $h < h_c$, the population may still become extinct if the initial population X_0 is below some critical level X_c, perhaps due to an ecological disaster. What is this critical initial value X_c?

Section-III

Q4. ***Attempt any four of the following:*** *(5+5+5+5)*

(a) Solve the initial value problem $y^{(3)} + 3y'' - 10y' = 0;\ y(0) = 7, y'(0) = 0, y''(0) = 70.$

(b) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' + y = tanx.$$

(c) Use the method of undetermined coefficients to solve the differential equation

$$y'' + 4y = 3x^3.$$

(d) A mass of 3kg is attached to the end of a spring that is stretched 20 cm by a force of 15N. It is set in motion with initial position $x_0 = 0$ and initial velocity v_0 = -10 m/s. Find the amplitude, period and frequency of the resulting motion.

Sl No. 17 Q.P. 3657 (6)

2019

Unique Paper Code : 235203

Name of the Paper : MAHT 202 – Analysis - II

Name of Course : ~~Semester Mode~~ B.Sc. (H) Mathematics

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

All questions are compulsory.

Attempt any **three** parts from each question.

1 (a) Show that $\lim_{x\to c} \frac{1}{x} = \frac{1}{c}$; $c \in \mathbb{R}, c > 0$ using $\varepsilon - \delta$ definition of limit.

(b) If $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$ has a limit at $c \in A$, then prove that f is bounded in some neighbourhood of c.

(c) Using sequential criteria, show that $\lim_{x\to 0} \frac{1}{x}$ does not exist.

(d) Define limit at infinity and calculate the $\lim_{x\to\infty} \frac{1}{x^2}$. (5,5,5, 5)

2 (a) Use definition of continuity to show the function

$$f(x) = \begin{cases} 1; & x \text{ is rational} \\ 0; & x \text{ is irrational} \end{cases}$$

is not continuous at any $x \in \mathbb{R}$.

(b) Prove that a continuous function on a closed and bounded interval is bounded.

(c) State Bolzano's Intermediate Value Theorem for continuous functions. Use it to show $xe^{x^2} = 10$ for some $x \in [0,2]$.

(d) Let $f(x)$ and $g(x)$ be continuous functions on $A \subseteq \mathbb{R}$ such that $g(x) \neq 0$ for all $x \in A$. Show that the function $f(x)/g(x)$ is continuous on A. (5,5,5,5)

3 (a) Show that the function $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[1, \infty)$ but not uniformly continuous on $(0, \infty)$.

(b) Prove that if a function $f(x)$ is uniformly continuous on A, then $f(x)$ is continuous on A. Is the converse true? Justify.

(c) Let $f(x) = 1 + x^3; x \in (1,2]$. Find f^{-1}. Is f^{-1} continous?

(d) State and illustrate the chain rule for differentiation of functions. (5,5,5,5)

4 (a) Let $f: \mathbb{R} \to \mathbb{R}$ be a function given by $f(x) = x^3 - 3x + 5$. Find the interval where the function is decreasing and the interval where the function is increasing.

(b) State and prove the Darboux's Theorem for a differentiable function.

(c) Use the Mean Value Theorem to prove that $|\sin(x) - \sin(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(d) Find the absolute maxima and minima for the function given by $f(x) = x|x^2 - 12|; x \in [-2,3]$. (5,5,5,5)

5 (a) Use Taylor's Theorem to show that $1 - \frac{x^2}{2} \leq \cos(x)$ for all $x \in \mathbb{R}$.

(b) Find the Taylor series expansion of e^{2x} about the point 0.

(c) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{(n+1)(n+2)}.$$

(d) Check if the function $f(x) = x^2 - 2x + 1; x \in [0,2]$ is convex. (5,5,5,5)

~~Set A~~ (7) 2019

Sr. No. of Question Paper : 3658

Unique Paper Code : 235204

Name of the Paper : Probability & Statistics-MAHT 203

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II

Duration : 3 Hours

Maximum Marks: 75

Instructions for Candidates

a) Write your Roll No. on the top immediately on receipt of this question paper.
b) In all there are **six** questions.
c) Question No. 1 is compulsory and it contains five parts of **3** marks each.
d) In Question No. **2** to **6**, attempt any **two** parts from **three** parts. Each part carries **6** marks.
e) Use of scientific calculator is allowed.

1. (i) If C_1 and C_2 are events in a sample space S . Then prove that

$$P(C_1 \cap C_2) \geq P(C_1) + P(C_2) - 1.$$

(ii) A bowl contains 16 chips, of which 6 are red, 7 are white and 3 are blue. If 4 chips are taken at random and without replacement, find the probability that : (a) each of the 4 chips is red: (b) None of 4 chips is red.

(iii) Let X be a random variable with cdf F_X.

Then for a $< b$, $P(a < X \leq b) = F_X(b) - F_X(a)$.

(iv) Let X has a negative exponential distribution with parameterλ. If $P(X \leq 1) = P(X > 1)$, what is the variance of X.

(v) Show that if a random variable has a uniform density with the parameters α and β, the probability that it will take on a value less than $\alpha + p(\beta - \alpha)$ is equal to p.

2. (a) State and prove Boole's inequality.

(b) Cast a dice a number of independent times till a six appears on the up face of the dice

(i) Find the pmf p(x) of X, the number of casts needed to obtain first six

(ii) Show that $\sum_{x=1}^{\infty} p(x) = 1$

(c)) Let a random variable X has pmf given by $p(x) = \begin{cases} \frac{1}{3} & ; x = -1,0,1 \\ 0 ; & otherwise \end{cases}$

Find the cdf F(x) of X.

3. (a) Let the pmf $p(x)$ be positive at $x = -1,0,1$ and zero elsewhere

(i) If $p(0) = \frac{1}{4}$. Find $E(X^2)$

(ii)) If $p(0) = \frac{1}{4}$ and $E(X) = \frac{1}{4}$. Determine p(-1) and p(1)

(b) Prove that moment generating function of Poisson distribution is given by $M_X(t) = e^{\lambda(e^t - 1)}$. Hence find its mean and variance.

(c) Let X have pdf $f(x) = 3x^2 : 0 < x < 1$, zero elsewhere. Consider a random rectangle whose sides are X and (1-X). Determine the expected value of the area of the rectangle.

4. (a) Let X has the pdf given by $f(x) = \begin{cases} cx^3 ; & 0 < x < 2 \\ 0 & otherwise \end{cases}$

Find the

(i) Constant c

(ii) $P(\frac{1}{4} < X < 1)$

(b) Prove that the mean and variance of the Uniform distribution are given by

$$\mu = \frac{\alpha+\beta}{2}, \qquad \sigma^2 = \frac{(\beta-\alpha)^2}{12}$$

(c) Prove that the moment generating function of Normal distribution is given by

$$M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2 t^2}$$

5.(a) The joint pdf of random variables X and Y is given by

$$f(x,y) = \begin{cases} x+y; & 0<x<1, 0<y<1 \\ 0; & otherwise \end{cases}$$

Find the conditional mean and variance of Y, given X= x, $0 < x < 1$.

(b) Define the independence of two variables X_1 and X_2. Suppose X_1, X_2 have the joint cdf $F(x_1, x_2)$ and marginal cdfs $F_1(x_1)$ and $F_2(x_2)$ respectively. Show that the variables X_1 and X_2 are independent if and only if $F(x_1, x_2) = F_1(x_1)F_2(x_2)$ for all $(x_1, x_2) \in R^2$.

(c) Suppose the random variables X and Y have the joint density given by

$$f(x,y) = \begin{cases} xe^{-x(1+y)}; & x>0,\ y>0 \\ 0 & otherwise \end{cases}$$

Find the regression equation $\mu_{Y|X}$ of Y on X. Also sketch the regression curve.

6. (a) State and Prove Chapman-Kolmogorov's equations.

(b) Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500 :

(i) What can be said about the probability that this week's production will be atleast 1000?

(ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600?

(c) State and prove central limit theorem for independent, identically distributed random variables with finite variance.

8 2019

This question paper contains ..2.. pages

Roll No.

S. No. of Question Paper : 6366

Unique Paper Code : **203261**

Name of the Paper : **English Language**

Name of the Course : **English for B.A.(Hons.)/B.Sc.(Hons.) Math./B.Sc. Math SC**

Semester : **II/IV**

Duration: **3 Hours**

Maximum Marks: **75**

(Write your Roll No. on the top immediately on receipt of this question paper)

Question No. **1** is compulsory. Answer any *three* parts of Question No. **1**.

Answer any *three* from the rest of questions **2** to **7**.

1. Answer any *three* of the following: 3x10

(a) O what is that sound which so thrills the ear
Down in the valley drumming, drumming?
Only the scarlet soldiers, dear,
The soldiers coming.

(i) Name the poem and the poet. 3

(ii) Who are the two speakers? 3

(iii) How does war affect human society? 4

(b) After the cruelest of winters, the house still stood. It was pale, washed clean by elements gone wild, and here and there a shutter dangled from a broken hinge.

(i) Identify the source of these lines. 3

(ii) Explain: "elements gone wild." 3

(iii) What is the condition of the house? 4

(c) When will the bell ring, and end this weariness?
How long have they tugged the leash, and strained apart,
My pack of unruly hounds! ...

(i) Identify Name the poem and the poet. 3
(ii) Why does the speaker feel weary? 3
(iii) Comment on the tone of these lines. 4

(d) So you think you can get away with it because you're a woman. A creature of poetry and romance, huh?

(i) Name the play and the playwright. 3
(ii) Who is the speaker? Why has he come to meet the lady? 3
(iii) What is meant by "a creature of poetry and romance"? 4

(e) The other little girls and boys were quite at ease—they had been at parties before. They were not afraid to talk or laugh and their throats didn't become whispery and hoarse when anyone asked them a question. I became more and more miserable with each passing moment.

(i) Identify the source of these lines. 3
(ii) What sort of party was it? Had the speaker attended such parties before? 3
(iii) While the other children were at ease, the speaker was miserable. Why? 4

(f) If all the beasts were gone, men would die from great loneliness of spirit; for whatever happens to the beasts also happens to man. All things are connected.

(i) Identify the source of these lines. 3
(ii) Explain the significance of the last sentence. 3
(iii) Discuss the speaker's concern for preserving the environment. 4

2. What is the central idea of the poem *O What Is That Sound*? 15

3. In what ways was WG Grace a great cricketer? 15

4. Phillips succeeds in building the A-bomb. What does it prove about the process and procurement of materials for bomb making? 15

5. What are Aurangzeb's views on education? Discuss. 15

6. Discuss the story 'Stench of Kerosene' as a tragic love story. 15

7. Write a character sketch of Sulbha from the play *Leaving Home*. 15

A

(This Question Paper contains 2 printed pages)

Your Roll No. 2019

आपका अनुक्रमांक ....

S. No of Question Paper 6419

प्रश्न पत्र का क्रमांक ....

(9)

Unique Code No. : 232671

यूनिक कोड : 232671

Name of the Course : B.SC. (Maths) Political Science

पाठ्यक्रम का नाम : बी.एससी. (गणित)

Title of Paper : Citizenship in a Globalizing World

Semester/Annual : Part-II

सेमेस्टर/वार्षिक : भाग– II

Time: 3 Hours

समय : 3 घण्टे

Maximum Marks: 75

पूर्णांक : 75

(Write your Roll No. on the top immediately on receipt of this question paper)

(इस प्रश्न–पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।)

Note : Answers may be written either in English or Hindi but the same medium should be followed throughout the paper.

इस प्रश्नपत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Attempt any **four** questions.

All questions carry equal marks.

किन्हीं चार प्रश्नों के उत्तर दीजिये।

सभी प्रश्नों के अंक समान हैं।

1. What is the liberal idea of citizenship? How is it different from the communitarian view?

नागरिकता की उदारवादी अवधारणा क्या है। यह समुदायवादी दृष्टिकोण से अलग कैसे है।

2. Examine the idea of multiculturalism and its implications for citizenship.

बहुसंस्कृतिवाद की और उसके नागरिकता पर होने वालो प्रभावों का मूल्यांकन कीजिए।

3. What is globalization? Discuss its impact on the nation-state.

वैश्वीकरण क्या है? इसका प्रभाव राष्ट्र–राज्य पर क्या रहा है?

4 Discuss the impact of migration on citizenship in states in our times.
वर्तमान में राज्यों में प्रवासन का नागरिकता पर प्रभाव की विवेचना कीजिए।

5 What is cosmopolitanism? How is cosmopolitanism different from nationalism?
सर्वदेशीयवाद क्या है? सर्वदेशीयवाद और राष्ट्रवाद में क्या अंतर है?

6 Why is citizenship important in our globalizing world?
वैश्वीकरण की दौर में नागरिकता को महत्वपूर्ण क्यों माना जाता है?

7 Discuss citizenship and diversity in the context of India and Canada.
कनाडा और भारत के संदर्भ में विभिन्नता और नागरिकता की विवेचना कीजिए।

8 Write Short notes on any **two** of the following:
निम्नलिखित में से किन्हीं दो पर संक्षिप्त टिप्पणी लिखिए

a. Global Justice
वैश्विक न्याय

b. Human Rights
मानव अधिकार

c. Direct Democracy
प्रत्यक्ष प्रजातंत्र

d. Feminist views on citizenship
नागरिकता पर नारीवादी दृष्टिकोण